# A Brief Technical Note on the Sensitivity of Zero-Vibration Shaper in Crane Systems

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**Abstract:** This technical note examines the sensitivity of a finite-duration acceleration command obtained using an undamped zero-vibration (ZV) shaper for a linearized overhead crane. A constant-acceleration maneuver lasting one nominal pendulum period is considered, with the acceleration and duration held fixed when the actual system differs from the design model. Exact terminal responses are derived for a constant cable-length error and for unmodeled equivalent viscous damping. The residual amplitude caused by a small relative cable-length error is proportional to the magnitude of that error, whereas the initial envelope of the damped free response is proportional to the damping ratio. Their leading-order ratio is twice the damping ratio divided by the absolute relative cable-length error. This comparison concerns two separate perturbations under the same prescribed maneuver; it is not a universal ranking of friction and cable-length effects. The analysis distinguishes the mathematical impulse filter from the physical command and clarifies the definition of residual vibration in a damped system. It provides a compact basis for assessing model mismatch while identifying the additional considerations required for nonlinear friction, aerodynamic loading, actuator dynamics, and time-varying cable length.

**Keywords:** input shaping; zero vibration; overhead crane; cable-length uncertainty; equivalent viscous damping; residual vibration

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## 1. Introduction

Input shaping is a feedforward approach to reducing the residual vibration generated by a prescribed motion command. Its effectiveness depends on the relationship between the command timing and the oscillatory modes of the controlled system. For overhead cranes, Alghanim et al. developed an input-shaping control scheme that provides a basis for studying command design and payload oscillation suppression [1]. Mohammed et al. subsequently considered trajectory control with nonzero initial states [2]. These studies address related but distinct operating conditions: cancellation of command-induced vibration from an initially stationary payload does not by itself ensure cancellation of pre-existing oscillations.

The wider development of input shaping includes zero-vibration-and-derivative (ZVD), extra-insensitive (EI), multimode, constrained, and adaptive approaches. Yang et al. review input shaping within the broader field of data-driven vibration control and discuss design, optimization, and engineering applications [3]. In crane-specific work, harmonic command design has been investigated for performance optimization [4], and modified shaping strategies have been developed for operation over a range of cable lengths [5]. A minimalistic constant-input strategy has also been examined for a double-pendulum crane model [6]. Together, these contributions motivate careful attention to both the chosen command and the assumptions used to predict its residual vibration.

Cable length is a particularly important parameter because it determines the natural frequency of the simple-pendulum approximation. Other departures from a nominal model can also alter the terminal payload state. Examples include damping, hoisting motion, actuator tracking errors, and aerodynamic loading. However, these effects should not be

combined indiscriminately: a constant frequency error, a dissipative force, and an externally imposed disturbance enter the dynamics in different ways. A useful sensitivity comparison must identify what changes in the plant and what remains fixed in the command.

This note examines two separate departures from a nominal undamped model: a small, constant relative cable-length error and a small equivalent viscous damping ratio. The prescribed acceleration and maneuver duration are held fixed in both comparisons. Exact terminal states are derived before obtaining leading-order expressions for residual vibration. The purpose is to clarify the sensitivity of this particular ZV-shaped command, rather than to introduce a new robust shaper or to establish experimentally validated performance. The distinction between an undamped oscillation amplitude and a damped free-response envelope is retained throughout the derivation.

## 2. Crane Model and Nominal Command

### *2.1. Model Assumptions*

Consider a payload suspended from a rigidly prescribed trolley by an inextensible cable of constant length $L > 0$. The payload is represented as a point mass, the swing angle is small, and the nominal model contains no damping. Let $u(t)$ denote trolley displacement and let the angular sign convention be chosen so that positive trolley acceleration produces a positive forcing term. The linearized equation is

$$\ddot{\theta} + \omega^2 \theta = \frac{\ddot{u}}{L}, \qquad \omega = \sqrt{\frac{g}{L}}, \tag{1}$$

where $g$ is gravitational acceleration. Reversing the angular sign convention reverses the angular response but leaves the residual amplitude unchanged. The initial payload conditions are $\theta(0) = 0$ and $\dot{\theta}(0) = 0$. Hoisting, cable elasticity, additional pendulum modes, and actuator dynamics are excluded from this model.

The maneuver increases trolley speed from zero to a specified value $v_c > 0$. It is therefore a rest-to-constant-speed maneuver, not a complete rest-to-rest transfer. Define the nominal duration $\tau$ and acceleration magnitude $A$ by

$$\tau = \frac{2\pi}{\omega}, \qquad A = \frac{v_c}{\tau} = \frac{\omega v_c}{2\pi}, \qquad \ddot{u}(t) = \begin{cases} A, & 0 < t < \tau, \\ 0, & t > \tau. \end{cases} \tag{2}$$

The acceleration is zero before the maneuver. Its time integral is $v_c$, so the terminal trolley speed is prescribed independently of the subsequent payload oscillation. The acceleration has finite jumps at its start and end; it is bounded, but it is not a smooth or jerk-limited command.

### *2.2. Relation to the ZV Shaper*

An input shaper is a mathematical filter whose impulses define weighted and delayed copies of a reference command. Those impulses do not imply that ideal impulsive forces must be applied directly to the physical trolley. For the nominal undamped mode, a two-impulse ZV filter has equal weights of one half and a separation of $T = \pi/\omega$. Applying it to a rectangular reference acceleration of magnitude $2A$ and duration $T$ gives two adjacent acceleration segments, each of magnitude $A$ and duration $T$. Their union is precisely the full-period constant-acceleration command in Eq. (2).

This construction explains the connection to ZV shaping without treating finite-duration steps as a universal replacement for impulse-based filtering. It also identifies the scope of the sensitivity calculation: the response belongs to the specified filter–reference combination. A different reference command, shaper duration, or damping-aware filter can yield a different residual response. Practical realizations must additionally respect acceleration and jerk limits, and any smoothing of the command should be included in the response calculation.

For the zero initial conditions, the nominal forced response during the maneuver is

$$\theta(t) = B(1 - \cos\omega t), \qquad \dot{\theta}(t) = B\omega \sin\omega t, \qquad B = \frac{A}{g} = \frac{v_c}{2\pi\omega L}. \tag{3}$$

Substitution of $t = \tau$ into Eq. (3) gives zero terminal angle and angular velocity. Once the acceleration ends, the nominal payload therefore remains unexcited while the trolley continues at constant speed. This cancellation follows from the chosen full-period duration and the assumed linear, undamped dynamics; it is not a statement of robustness to uncertain parameters.

**3. Sensitivity to Cable-Length Error**

Let the actual cable length be $\bar{L} = L(1 + a)$, where $a > -1$ and $|a| \ll 1$. The actual natural frequency is $\bar{\omega} = \omega/\sqrt{1 + a}$. Both $A$ and $\tau$ remain at their nominal values in Eq. (2); changing them with the actual length would define a retuned maneuver rather than a sensitivity test. The perturbed undamped response at the nominal stopping time is

$$\theta_a(\tau) = \frac{A}{\bar{L}\bar{\omega}^2}[1 - \cos(\bar{\omega}\tau)] = B[1 - \cos(\bar{\omega}\tau)], \tag{4}$$

$$\dot{\theta}_a(\tau) = B\bar{\omega}\sin(\bar{\omega}\tau), \qquad \bar{\omega}\tau = \frac{2\pi}{\sqrt{1 + a}}. \tag{5}$$

The identity $\bar{L}\bar{\omega}^2 = g$ makes the displacement prefactor $B = A/g$ independent of the length error. This is essential when the imposed acceleration is unchanged. Replacing every nominal frequency and length in Eq. (3) without preserving the fixed acceleration would inadvertently change the input and produce a different exact terminal state.

After the command ends, the actual undamped system undergoes free oscillation. Its angular amplitude is

$$R_a = \sqrt{\theta_a^2(\tau) + \frac{\dot{\theta}_a^2(\tau)}{\bar{\omega}^2}} = 2B\left|\sin\left(\frac{\pi}{\sqrt{1 + a}}\right)\right|. \tag{6}$$

Thus, the exact residual amplitude has a compact trigonometric form. It vanishes at the nominal length, but Eq. (6) also retains the full dependence on a constant length mismatch within the linear model. The absolute value is necessary because an amplitude cannot be negative, irrespective of whether the actual cable is longer or shorter than its nominal value.

Expanding the terminal state about $a = 0$ gives

$$\theta_a(\tau) = \frac{B\pi^2}{2}a^2 + O(a^3), \qquad \frac{\dot{\theta}_a(\tau)}{\bar{\omega}} = -B\pi a + O(a^2). \tag{7}$$

The terminal angle is second order, whereas the normalized terminal angular velocity is first order. Consequently,

$$R_a = \frac{v_c}{2\omega L}|a| + O\left(\frac{v_c}{\omega L}a^2\right). \tag{8}$$

Eq. (8) shows why a small terminal angle alone is insufficient evidence of successful vibration cancellation: a nonzero terminal angular velocity can generate a first-order residual oscillation. The leading term is symmetric in the sign of the cable-length error, although the exact finite-error response need not be symmetric. Terms obtained by retaining selected powers inside a square root should not be interpreted as a controlled higher-order expansion unless all contributions of the same order are included.

## 4. Sensitivity to Unmodeled Viscous Damping

### *4.1. Terminal Response*

For the second comparison, retain the nominal cable length and introduce equivalent viscous damping with $0 \leq \zeta < 1$. The commanded acceleration and duration are again unchanged. The equation of motion becomes

$$\ddot{\theta} + 2\zeta\omega\dot{\theta} + \omega^2\theta = \frac{\ddot{u}}{L}, \qquad \omega_d = \omega\sqrt{1-\zeta^2}. \tag{9}$$

Here, $\zeta$ represents damping of the payload swing mode. It is not automatically an equivalent representation of trolley stiction or motor friction, particularly when trolley acceleration is prescribed independently. Likewise, nonlinear aerodynamic drag can be represented by a constant viscous coefficient only within a justified operating approximation. The calculation therefore concerns the specified linear damping model rather than every possible frictional or aerodynamic disturbance.

With zero initial conditions and constant acceleration $A$, the response is

$$\theta(t) = B\left[1 - e^{-\zeta\omega t}\left(\cos\omega_d t + \frac{\zeta\omega}{\omega_d}\sin\omega_d t\right)\right], \tag{10}$$

$$\dot{\theta}(t) = B\frac{\omega^2}{\omega_d}e^{-\zeta\omega t}\sin\omega_d t. \tag{11}$$

For compact notation, set $q = \sqrt{1-\zeta^2}$, $E = e^{-2\pi\zeta}$, and $\phi = 2\pi q$. Evaluating Eqs. (10) and (11) at $\tau$ yields

$$\theta_\tau = B\left[1 - E\left(\cos\phi + \frac{\zeta}{q}\sin\phi\right)\right], \qquad \dot{\theta}_\tau = B\frac{\omega}{q}E\sin\phi. \tag{12}$$

In general, this terminal state is nonzero. Damping changes both the transient weighting and the oscillation frequency, so the cancellation designed for an undamped mode is no longer exact. This loss of cancellation does not mean that damping destabilizes the system: the subsequent free response still decays.

### *4.2. Residual Envelope and Leading-Order Sensitivity*

The residual response must be defined consistently with the damped dynamics. For elapsed time $s = t - \tau \geq 0$ after acceleration ceases, the free response is

$$\theta(\tau + s) = e^{-\zeta\omega s}\left[\theta_\tau\cos\omega_d s + \frac{\dot{\theta}_\tau + \zeta\omega\theta_\tau}{\omega_d}\sin\omega_d s\right]. \tag{13}$$

Its initial envelope coefficient is therefore

$$R_\zeta = \sqrt{\theta_\tau^2 + \left(\frac{\dot{\theta}_\tau + \zeta\omega\theta_\tau}{\omega_d}\right)^2} = \frac{B}{q}\sqrt{1 + E^2 - 2E\cos\phi}. \tag{14}$$

The envelope at elapsed time $s$ is $R_\zeta e^{-\zeta\omega s}$. This coefficient bounds the absolute angular response but is not, in general, its attained maximum after the maneuver. The undamped expression in Eq. (6) cannot be used as an exact damped amplitude because the phase coefficients in Eq. (13) contain both damping and the damped natural frequency.

For small nonnegative $\zeta$, the terminal-state expansion gives

$$\theta_\tau = 2\pi B\zeta + O(B\zeta^2), \qquad \frac{\dot{\theta}_\tau}{\omega} = -\pi B\zeta^2 + O(B\zeta^3). \tag{15}$$

Using these expansions in Eq. (14) gives

$$R_\zeta = \frac{v_c}{\omega L}\zeta + O\Big(\frac{v_c}{\omega L}\zeta^2\Big). \tag{16}$$

Unlike the length-error case, the leading terminal mismatch is in angle. The resulting first-order scaling is nevertheless expressed using the same characteristic angular scale $v_c/(\omega L)$. This common scale allows a direct local comparison while preserving the different physical meanings of the two residual measures.

## 5. Comparison and Practical Implications

For a nonzero cable-length error and small positive damping, Eqs. (8) and (16) yield

$$\frac{R_\zeta}{R_a} = \frac{2\zeta}{|a|}[1 + O(\zeta) + O(|a|)] \approx \frac{2\zeta}{|a|}. \tag{17}$$

This is a leading-order comparison between two separate perturbed systems subjected to the same nominal command. It is undefined at $a = 0$ and should not be written as an exact finite-parameter identity. Equal numerical values of $\zeta$ and $|a|$ produce an approximately twofold difference in the initial residual measures, but the two parameters describe different physical departures from the nominal model.

Eq. (17) identifies a useful local comparison: unmodeled damping and cable-length mismatch give comparable initial residual measures when $2\zeta$ is approximately $|a|$. It does not imply that damping is always more detrimental, since the damped residual subsequently decays while the idealized undamped residual persists. Nor does it quantify a system in which both uncertainties act simultaneously. Such a calculation requires their joint terminal response rather than the addition of separately derived amplitudes.

The result also does not establish that every ZV implementation has the same sensitivity. A shaper designed using a known damping ratio can use damping-dependent weights and timing, while a different reference waveform changes the complete shaped command. More robust designs must be compared under stated timing, acceleration, and actuator constraints. The present calculation isolates one source of mismatch and therefore supplies a diagnostic reference rather than a universal performance ranking.

For crane applications, several extensions remain distinct from the model considered here. A cable length that changes during the maneuver introduces time-dependent dynamics rather than the constant parameter error used in Eqs. (4)–(8). Trolley friction and actuator delays can change the achieved acceleration, requiring the actual command-tracking dynamics to be modeled. Coulomb friction, stiction, and quadratic drag cannot generally be reduced to a constant viscous coefficient across arbitrary amplitudes and speeds. Cable flexibility and double-pendulum behavior introduce additional modes. These effects should be incorporated according to the intended operating regime before the local sensitivity estimates are used for design decisions.

## 6. Conclusion

For the specified full-period, constant-acceleration command, a small constant relative cable-length error produces a residual angular amplitude of $v_c|a|/(2\omega L)$ to leading order. Unmodeled equivalent viscous damping produces an initial free-response envelope coefficient of $v_c\zeta/(\omega L)$ to leading order. Their ratio is approximately $2\zeta/|a|$ when both perturbations are small and the cable-length error is nonzero. The nominal acceleration and duration must remain fixed for this comparison to represent sensitivity rather than retuning.

The derivation shows that length mismatch primarily creates a first-order terminal angular-velocity error, whereas small unmodeled damping primarily creates a first-order terminal angular-displacement error. Both can prevent exact nominal cancellation, but damping also causes the subsequent oscillation to decay. These results support explicit treatment of damping assumptions and command implementation in crane input-shaping

studies. They apply to the stated linear, single-mode model and do not replace validation for nonlinear friction, actuator tracking errors, hoisting motion, or additional flexible modes.

### Competing Interests

The author declares no competing interests.

### Data Availability

This technical note presents theoretical derivations and does not report an experimental dataset. The governing equations, prescribed command, exact responses, and asymptotic expressions needed to reproduce the analysis are provided in the text.

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